

Quantifying Chiral Magnetic Effect from Anomalous-Viscous Fluid Dynamics

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Chiral Magnetic Effect (CME) is the macroscopic manifestation of the fundamental chiral anomaly in a many-body system of chiral fermions, and emerges as anomalous transport current in the fluid dynamics framework. Experimental observation of CME is of great interest and has been reported in Dirac and Weyl semimetals. Significant efforts have also been made to search for CME in heavy ion collisions. Encouraging evidence of CME-induced charge separation in those collisions has been reported, albeit with ambiguity due to background contamination. Crucial for addressing such issue, is the need of quantitative predictions for CME signal with sophisticated modelings. In this paper we develop such a tool, the Anomalous Viscous Fluid Dynamics (AVFD) framework, which simulates the evolution of fermion currents in QGP on top of the data-validated VISHNU bulk hydrodynamic flow. With realistic initial conditions and magnetic field lifetime, the AVFD-predicted CME signal could be quantitatively consistent with measured charge separation in 200A GeV AuAu collisions.

Introduction.— The importance of electricity for modern society cannot be overemphasized. From the physics point of view, lies at the heart of electricity is the conducting transport (of electric charge carriers). In normal materials, conducting transport generates an electric current \vec{J}_Q along the electric field \vec{E} (or voltage) applied to the system. This can be described by the usual Ohm's law $\vec{J}_Q = \sigma_e \vec{E}$ where the conductivity σ_e arises from competition between “ordered” electric force and “disordered” thermal scatterings, henceforth involving dissipation and typically dependent upon specific dynamics of the system. More recently there have been significant interests, from both high energy and condensed matter physics communities, in a new category of *anomalous chiral transport* in quantum materials containing chiral fermions. A notable example is the Chiral Magnetic Effect (CME) [1–5] — the generation of an electric current \vec{J}_Q along the *magnetic field* \vec{B} applied to the system, i.e.

$$\vec{J}_Q = \sigma_5 \vec{B} \quad (1)$$

where $\sigma_5 = C_A \mu_5$ is the chiral magnetic conductivity, expressed in terms of the chiral chemical potential μ_5 that quantifies the imbalance between fermions of opposite (right-handed, RH versus left-handed, LH) chirality.

The σ_5 has two remarkable features that make it markedly different from the normal conductivity σ . First, the coefficient C_A takes a *universal value* of $Q_f^2/(4\pi^2)$ (for each species of RH or LH fermions with electric charge Q_f) from non-interacting cases to extremely strongly coupled cases [5–8]. In fact, it is entirely dictated by universal chiral anomaly coefficient, and the CME is really just the macroscopic manifestation of the fundamental quantum anomaly in a many-body setting. Second, the σ_5 is time-reversal even [9] which implies the non-dissipative nature of the underlying transport process

that leads to the CME current in (1).

Given the magnificent physics embodied in the Chiral Magnetic Effect, it is of the utmost interest to search for its manifestation in real-world materials. So far two types of systems for experimental detection of CME have been enthusiastically investigated. One is the so-called Dirac and Weyl semimetals where electronic states emerge as effective chiral fermions and exhibit chiral anomaly [10, 11]. Discoveries of CME were reported in those systems [12–15]. The other is the quark-gluon plasma (QGP), which is the deconfined form of nuclear matter at very high temperatures $T \sim$ trillion degrees, consisting of approximately massless light quarks as chiral fermions. Such a new form of hot matter once filled the whole universe and is now (re)created in relativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). Dedicated searches for potential CME signals have been ongoing at RHIC and the LHC [16–21], with encouraging evidences reported through measuring the charge separation signal induced by the CME current (1). The interpretation of these data however suffer from backgrounds arising from the complicated environment in a heavy ion collision (see reviews and references in e.g. [22–25]). Currently the most pressing challenge for the search of CME in heavy ion collisions is to clearly separate background contributions from the desired signal. A mandatory and critically needed step, is to develop state-of-the-art modeling tools that can quantify CME contribution in a realistic heavy ion collision environment. In this Letter we present such a tool, the Anomalous Viscous Fluid Dynamics (AVFD) framework, which simulates the evolution of chiral fermion currents in the QGP on top of the data-validated VISHNU bulk hydrodynamic evolution for heavy ion collisions. We demonstrate the special features pertaining to anomalous transport in this frame-

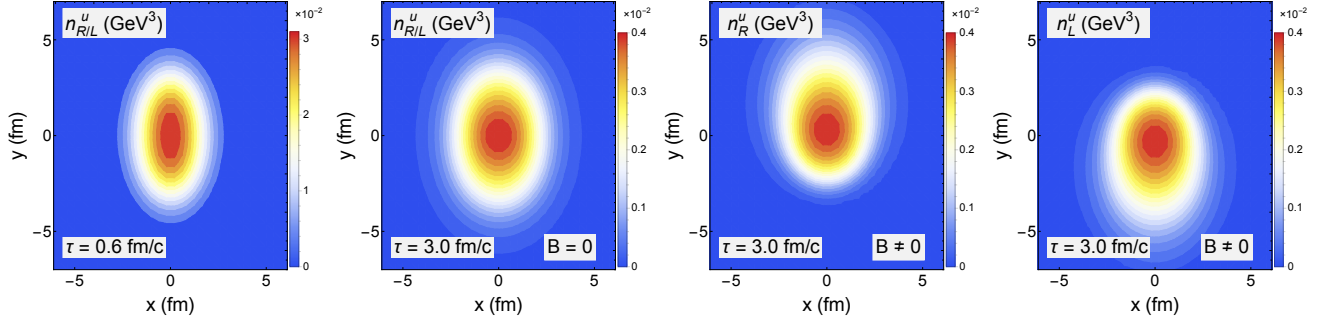


FIG. 1: (color online) The evolution of u-flavor densities via solving AVFD equations from the same initial charge density distribution (for either RH or LH) at $\tau_0 = 0.60 \text{ fm/c}$ (left most panel) in three different cases: (a) (second left panel) for either RH or LH density at $\tau = 3.00 \text{ fm/c}$ with zero magnetic field $B \rightarrow 0$ which implies no anomalous chiral transport; (b) (second right panel) for RH density at $\tau = 3.00 \text{ fm/c}$ with nonzero B field along positive y-axis; (c) (most right panel) for LH density at $\tau = 3.00 \text{ fm/c}$ with nonzero B field along positive y-axis.

work and quantify the CME-induced charge separation signal for comparison with available experimental data.

The Anomalous-Viscous Fluid Dynamics.—Fluid dynamics provides a universal description of macroscopic systems under the large scale and long time limit, and are essentially conservation laws (i.e. for energy, momentum, charged currents) arising from symmetries in microscopic dynamics. For a fluid of chiral fermions, the microscopic chiral anomaly is a sort of “half symmetry” and how it arises in macroscopic fluid dynamics is an interesting and nontrivial question. As answered in [6], the constituent relation for the fermion currents is required by the second law of thermal dynamics to include anomalous terms corresponding to the CME current and a similar chiral vortical current. Based on such finding, let us then develop a simulation framework, focusing on describing anomalous chiral transport in heavy ion collisions at very high beam energy (e.g. top RHIC energy and above). The bulk evolution in such collisions is well described by boost-invariant 2+1D 2nd-order viscous hydrodynamics (e.g. VISHNU simulations [26]) where net charge densities are small enough and typically neglected without much influence on bulk evolution. However to study the CME, one needs to accurately account for the evolution of fermion currents. Our approach is to solve the following fluid dynamical equations for the chiral fermion currents (RH and LH currents for u and d flavors respectively) as perturbations on top of the bulk fluid evolution:

$$\hat{D}_\mu J_{\chi,f}^\mu = \chi \frac{N_c Q_f^2}{4\pi^2} E_\mu B^\mu \quad (2)$$

$$J_{\chi,f}^\mu = n_{\chi,f} u^\mu + \nu_{\chi,f}^\mu + \chi \frac{N_c Q_f}{4\pi^2} \mu_{\chi,f} B^\mu \quad (3)$$

$$\Delta_\nu^\mu \hat{d}(\nu_{\chi,f}^\nu) = -\frac{1}{\tau_r} \left[(\nu_{\chi,f}^\mu) - (\nu_{\chi,f}^\mu)_{NS} \right] \quad (4)$$

$$(\nu_{\chi,f}^\mu)_{NS} = \frac{\sigma}{2} T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu_{\chi,f}}{T} \right) + \frac{\sigma}{2} Q_f E^\mu \quad (5)$$

where $\chi = \pm 1$ labels chirality for RH/LH currents and $f = u, d$ labels light quark flavor with respective electric charge Q_f and with color factor $N_c = 3$. The $E^\mu = F^{\mu\nu} u_\nu$ and $B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$ are external elec-

tromagnetic fields in fluid rest frame. The derivatives \hat{D}_μ is covariant derivative and $\hat{d} = u^\mu \hat{D}_\mu$, with projection operator $\Delta^{\mu\nu} = (g^{\mu\nu} - u^\mu u^\nu)$. In the above the fluid four-velocity field u^μ is determined by background bulk flow. Furthermore the (small) fermion densities $n_{\chi,f}$ and corresponding chemical potential $\mu_{\chi,f}$ are related by lattice-computed quark number susceptibilities $c_2^f(T)$. In the above equations the involved local temperature T as well as all other thermodynamic quantities are also from background bulk flow. It is worth emphasizing that the above framework treats the normal viscous currents $\nu_{\chi,f}^\mu$ at the second-order of gradient expansion by incorporating relaxation evolution toward Navier-Stokes form, thus in consistency with the background bulk flow also described by the 2nd-order viscous hydrodynamics. Two transport coefficients are involved: the normal diffusion coefficient σ and the relaxation time τ_r .

The most unique feature of the above Anomalous-Viscous Fluid Dynamics (AVFD) framework lies in the B -field driven anomalous current — the last term in Eq.(3) which distinguishes the left from the right with opposite sign. We demonstrate the effect of such chiral transport in Fig.1, by computing the evolution of u-flavor currents via solving AVFD equations from the same initial charge density distribution (for either RH or LH) at $\tau_0 = 0.60 \text{ fm/c}$ (left most panel) in three different cases: (a) (second left panel) for either RH or LH density at $\tau = 3.00 \text{ fm/c}$ with zero magnetic field $B \rightarrow 0$ which implies no anomalous chiral transport; (b) (second right panel) for RH density at $\tau = 3.00 \text{ fm/c}$ with nonzero B field along positive y-axis; (c) (right most panel) for LH density at $\tau = 3.00 \text{ fm/c}$ with nonzero B field along positive y-axis. In the case (a) the densities evolve only according to normal viscous transport i.e. charge diffusion which is identical for RH/LH densities and “up/down” symmetric. Under the presence of B field, additional transport occurs via anomalous currents along the B field direction, and RH/LH densities evolve in an asymmetric and opposite way. The effect of B -field driven anomalous chiral transport is evident from such a comparison.

CME-Induced Charge Separation.— With the AVFD

simulation tool introduced above, we are now ready to quantify the CME-induced charge separation signal under realistic conditions in heavy ion collisions. The way the charge separation arises in these collisions can be intuitively understood as follows: the \vec{B} field is along the out-of-plane (i.e. perpendicular to the reaction plane) direction, and the initial condition has nonzero axial charge density (i.e. imbalance between RH and LH), therefore one of the two patterns (the second right versus the right most) in Fig.1 would be dominant thus transporting excessive positively-charged u-quark density toward one end of the fireball along out-of-plane direction, with the opposite-direction transport for negatively charged d-quark densities, thus eventually causing a pattern of electric charge separation across the reaction plane.

Such a charge separation leads to a dipole term in the azimuthal distribution of produced charged hadrons:

$$\frac{dN^{ch}}{d\phi} \propto [1 \pm 2a_1^{ch} \sin \phi + \dots] \quad (6)$$

where ϕ is the azimuthal angle measured with respect to the reaction plane, and the $\pm a_1^{ch}$ for opposite charges respectively. From the preceding discussion, it is evident that the charge separation signal critically depends upon the magnetic field and initial axial charge, both of which are not theoretically well constrained so far.

For the magnetic field $\vec{B} = B(\tau)\hat{y}$ (with \hat{y} the event-wise out-of-plane direction), we use a plausible parametrization [27, 28]

$$B(\tau) = \frac{B_0}{1 + (\tau/\tau_B)^2} \quad (7)$$

The peak value B_0 (for each centrality) at the collision point has been well quantified with event-by-event simulations and we use the most realistic values from [29]. The lifetime τ_B however is poorly known [30–32]. Logically there are three possibilities: (a) τ_B is much longer than the initial start time of hydrodynamic evolution $\tau_0 = 0.6\text{fm}/c$ which appears unlikely; (b) τ_B is extremely short, $\tau_B \ll \tau_0$, in which case the anomalous chiral transport would have to occur in an out-of-equilibrium setting — we leave this possibility for future investigation; (c) τ_B is still short but comparable to τ_0 , i.e. $\tau_B \simeq \tau_0$ — for the present work we use this last assumption.

For the initial axial charge density arising from gluonic topological charge fluctuations, one could make following estimate based on the strong chromo-electromagnetic fields in the early-stage glasma, following a similar approach to that deployed in [33–35]:

$$\sqrt{\langle n_5^2 \rangle} \simeq \frac{Q_s^4 (\pi \rho_{tube}^2 \tau_0) \sqrt{N_{coll.}}}{16\pi^2 A_{overlap}} \quad (8)$$

In the above $\rho_{tube} \simeq 1\text{fm}$ is the transverse extension of glasma flux tube, $A_{overlap}$ is the geometric overlapping area of the two colliding nuclei, and $N_{coll.}$ the binary collision number for a given centrality. Such axial charge density depends most sensitively upon the saturation scale

Q_s , in the reasonable range of $Q_s^2 \simeq 1 \sim 1.5\text{GeV}^2$ for RHIC 200A GeV collisions [36–38].

In addition there are two important viscous transport parameters, the diffusion coefficient σ and the relaxation time τ_r , the values of which are not precisely determined yet, albeit narrowed down to certain plausible choice for the QGP in the relevant temperature regime. For the quantitative study of CME to a level of meaningful comparison with experimental data, it is crucial to understand the dependence of the anomalous CME signal on these normal viscous parameters and to characterize the associated theoretical uncertainty. This has not been possible in the few early attempts of anomalous transport modelings in the ideal hydrodynamic limit [27, 28, 35, 39]. The AVFD for the first time provides a tool to fully address such question. In Fig. 2 we show the computed charge separation signal a_1^{ch} for one centrality bin (30 ~ 60%) versus conductivity σ/T at various choices of $\tau_r T$ with T the temperature. Within a relatively wide range of values for σ/T and $\tau_r T$, the resulting a_1^{ch} varies from the ideal-hydro-limit (for $\sigma \rightarrow 0$) within about $\pm 30\%$ range. A “canonical choice” (see e.g. [40]) of $\sigma/T = 0.3$ and $\tau_r T = 0.5$, which will be used in our later computation, is indicated by the red blob, with the grey shaded band indicating a $\pm 5\%$ deviation in a_1^{ch} from this choice to give an idea of the theoretical uncertainty.

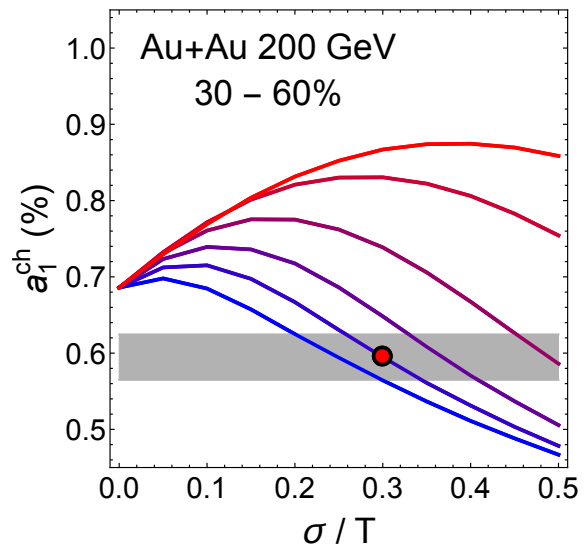


FIG. 2: (color online) The charge separation signal a_1^{ch} computed from AVFD for 30 ~ 60% centrality as a function of diffusion coefficient σ/T for given relaxation time $\tau_r T = 0.3, 0.5, 0.7, 1.0, 1.5, 2.0$ (from bottom/blue to top/red) respectively. The red blob indicates the result for $\sigma/T = 0.3$ and $\tau_r T = 0.5$, with the grey shaded band indicating a $\pm 5\%$ deviation in a_1^{ch} from this choice.

After the preceding discussions on the various aspects of the AVFD tool, let us now proceed to utilize this tool for quantifying CME toward comparison with available data. The measurement of a CME-induced charge separation is however tricky, as this dipole flips

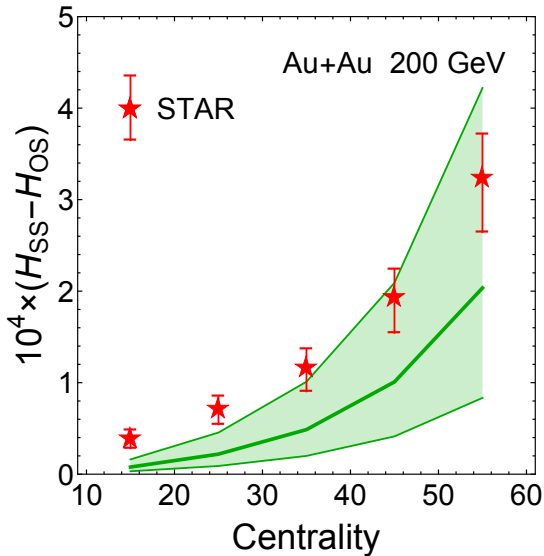


FIG. 3: (color online) The azimuthal correlation observable ($H_{SS} - H_{OS}$) for various centrality, computed from AVFD simulations and compared with STAR data [20], with the green band spanning the range of key parameter from $Q_s^2 = 1\text{GeV}^2$ (bottom edge) to $Q_s^2 = 1.5\text{GeV}^2$ (top edge).

its sign from event to event pending the sign of the initial axial charge arising from fluctuations, thus with a vanishing event-averaged mean value. What can be measured is its variance, through azimuthal correlations for same-sign (SS) and opposite-sign (OS) pairs of charged hadrons [16, 17, 41]. The so-called $\gamma_{SS/OS} \equiv \langle \cos(\phi_1 + \phi_2) \rangle$ observables measure a difference between the in-plane versus out-of-plane correlations and are indeed sensitive to potential CME contributions. They however suffer from considerable flow-driven background contributions that are not related to CME: see [22, 25] for reviews and references. A lot of efforts have been made, attempting to separate backgrounds from CME signals (see most recent discussions in e.g. [42–46]). One approach based on a two-component scenario [25, 42] was recently adopted by the STAR Collaboration to suppress backgrounds and extract the flow-independent part (referred to as $H_{SS/OS}$) [20]. We consider $H_{SS/OS}$ as our “best guess” thus far for potential CME signal to be compared with AVFD computations. Specifically a pure CME-induced charge separation will contribute as $(H_{SS} - H_{OS}) \rightarrow 2(a_1^{ch})^2$. The AVFD results for various centrality bins are presented in Fig. 3, with the green band spanning the range of key parameter Q_s^2 in $1 \sim 1.5\text{GeV}^2$ reflecting uncertainty in estimating initial axial charge (see Eq.(8)). Clearly the CME-induced correlation is very sensitive to the amount of initial axial charge density as controlled by Q_s^2 . The comparison with STAR data [20] shows very good agreement for the magnitude and centrality trend for choices with relatively large values of Q_s^2 . It though should be mentioned that the purpose for such comparison is *not* to state that CME explains data unambiguously. Given the current uncertainties both in theory (mainly on B-field lifetime and

initial axial charge) and in experiment (mainly potential residue backgrounds in the H -correlation [45, 46]), we feel it would be premature to draw any definitive conclusion yet. We’d rather emphasize the AVFD as a versatile tool for quantitatively studying the CME: once the theoretical and experimental uncertainties would be narrowed down in the future, the AVFD simulations will then readily allow a meaningful and conclusive comparison.

Summary and Discussions.— In summary, a new simulation tool — the Anomalous-Viscous Fluid Dynamics (AVFD) framework has been developed for quantifying the charge separation signal induced by Chiral Magnetic Effect in relativistic heavy ion collisions. This framework computes the evolution of chiral fermion currents in QGP on top of the data-validated VISHNU bulk hydrodynamic evolution. We find that, subject to current theoretical and experimental uncertainties, the AVFD-predicted CME signal with realistic initial conditions and magnetic field lifetime is quantitatively consistent with measurements from 200A GeV AuAu collisions at RHIC.

It is worth emphasizing again that the CME is a new type of macroscopic anomalous transport arising from microscopic anomaly in chiral matter. Given its observation in Dirac and Weyl semimetal systems in condensed matters experiments, it is now of extreme relevance and significance to unambiguously determine whether the same could be observed in an entirely different system i.e. the QGP, for the CME as a universal emergent phenomenon. The AVFD simulation frame will provide the necessary tool to address the pressing issues in the current hunt for the CME in heavy ion collisions.

We end by briefly mentioning a number of interesting problems that are being explored with this new tool. (a) One could study the possible anomalous transport of strangeness by comparing the charged kaon separation from AVFD computations with only u, d flavors and with three massless flavors, with the ratios a_1^K/a_1^{ch} about 0.4 in the former case while about 1.2 in the latter case. (b) If there is considerable CME transport occurring before the start of hydrodynamics, then such pre-hydro CME contribution can be incorporated into the AVFD framework as nontrivial initial conditions for the currents $J_{\chi,f}^\mu$ and it can be demonstrated that pre-hydro charge separation survives into final hadron observables. (c) Another important way to probe CME is through the so-called Chiral Magnetic Wave which leads to a splitting in the positive/negative pion elliptic flow [47, 48], and such an effect can also be quantitatively evaluated with the AVFD simulations. These problems, along with many other new studies enabled by the AVFD, will be reported in a long sequel elsewhere in the near future.

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